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EARLY *TIMAEUS* COMMENTARIES AND HELLENISTIC MUSICOLOGY

ANDREW BARKER

The fog into which Hellenistic work in harmonic science has disappeared is even thicker than that which has swallowed up the philosophical writings of the same period. No complete treatises survive; and if more than a very few refugees from their pages are still being harboured in later sources, most of them are well hidden, living out their twilight lives under assumed names, or none at all, and perhaps disguised behind the false beards of post-Hellenistic terminology. No one so far has made a concerted effort to flush them out; students of the subject have no Von Arnim or Long and Sedley to set them on course; any investigator rash enough to set foot in this area will be stumbling among unknown obstacles in the dark, and I am no exception. I cannot claim that the conclusions I reach in this paper are certain. But it is an attempt, however flawed, at sketching one of the pathways I think I have identified – possibly even a main thoroughfare – through the shadows of this invisible labyrinth.

I need to begin with a brief sketch of the terrain of harmonic science at the close of the fourth century. In the most general terms, harmonics is the study of the structures underlying musical melody, structures such as scales and patterns of attunement which provide the framework upon which melodies are hung. It examines their constituents and the ways in which they can legitimately be arranged, the relations between the various structures formed through these arrangements, the ways in which they can be transformed without collapsing into chaos, and so on; and it seeks to identify the overarching principles which govern the formation, interrelation and transformation of these structures, and which provide the basis for 'scientific' distinctions between genuinely musical patterns of organisation and merely random distributions of notes and intervals.

In Book 7 of the *Republic*, Plato identifies two quite distinct approaches to the subject, aside from his own idiosyncratic variant, and the distinction he draws is echoed – with modifications that need not much concern us – by other fourth-century writers, notably Aristotle and Aristoxenus.[1] Putting the various bits of evidence together we reach a picture of the following sort. One 'school' of harmonic science, labelled 'mathematical' or 'Pythagorean', begins from the thesis that relations between musical pitches are essentially quantitative, and in particular that the relation between two differently pitched notes is constituted by a ratio of numbers. Two notes an octave apart, for instance, stand in the ratio 2:1. The idea is not that we hear one note as twice as high as the other, whatever that would

[1] See Plato, *Rep.* 530c-531c, Aristotle, *An. Post.* 78b34-79a6, *Metaph.* 1053a12-17, Aristoxenus *El. Harm.* 32.18-31.

mean. It is encouraged in the first instance by observations of sounding objects; to take the simplest case, two lengths of string, one twice as long as the other but otherwise identical, will produce notes at an octave from one another. By itself that does not show that the notes or pitches themselves, like the string-lengths, can properly be said to stand in the ratio 2:1; but in the fourth century this approach was typically allied to a theory of the physical basis of pitch so designed as to allow that inference to be drawn. Sounds are movements in the air; differently pitched sounds, according to different hypotheses current in the period, travel at different speeds, or more or less vigorously, or are constituted by more or less rapid successions of micro-impulses.[2] We need not pursue the various alternatives. The point is that the representation of pitch-relations as numerical ratios purported to capture what these relations really amount to, independently of the way they present themselves to our hearing; and what they amount to are ratios. Questions about the proper ordering of musical relations then become transformed into questions about the formal organisation of numbers and ratios of numbers, and the principles defining systems of order that are musically correct are not specifically musical principles at all; they are principles of mathematical well-formedness or mathematically coherent organisation. Hence they specify forms of order that can be found outside music too. Music provides just one locus in which formal perfection can be manifested, but for the scientist or philosopher it has the advantage of being a matrix within which formal patterns and principles can be investigated fairly directly. Findings made here can then be transferred to obscurer domains, to specify the formula for a 'harmonious' integration of elements in a healthy body, for instance, or to illuminate the structure of the World Soul. Hence also this brand of harmonic science is regularly characterised as one based upon reasoning, rather than relying on the evidence of the ear.[3] A well formed musical structure is one that makes rational, mathematical sense; ideally it should be one that can be derived systematically from mathematical axioms.[4] It is not simply one that 'sounds right'; and the fact, if it is one, that it sounds wrong to our fallible mortal ears cannot legitimately be used to undermine its rationally excogitated credentials.

So much for the Pythagoreans; the opposing team's supporters sing from a very different song-sheet. They march under the banner of a sturdy empiricism, guided by the ear's perceptions of what is and is not musically admissible. Plato, who poses as a neutral observer but in fact has a seat in the stands very close to the Pythagorean benches, teases the exponents of this approach mercilessly,[5] and other evidence also suggests that in his time their methods and aspirations were indeed rather crude.[6] It was Aristotle's rehabilitation of the empirical

[2] Versions of these theories will be found, for instance, in Archytas frag. 1, Plato *Tim.* 79e-80b, Aristotle *De an.* 420a-b, [Euclid] *Sectio Canonis* 148.3-149.8 Jan.

[3] Its exponents 'dismiss perception as inaccurate and invent theoretical explanations', as Aristoxenus puts it with characteristic venom (*El. Harm.* 32.20-23). See also the passages of Ptolemaïs and Didymus quoted by Porphyry, *In Ptol. Harm.* 22.22-24.6, 25.3-26.5, 26.6-29.

[4] This systematic derivation is the project of the Euclidean *Sectio Canonis*.

[5] They are the target of Glaucon's patronizing amusement at *Rep.* 531a4-b1, and of Socrates' images of the musical torture-chamber in the lines immediately following (531b2-6). They are sharply distinguished from the Pythagoreans in 531b6-8.

[6] That, at any rate, is the picture presented in a series of cutting comments by Aristoxenus; see *El. Harm.* 2.7-3.1, 3.32, 5.6-27, 6.11-33, 7.3-5, 22-33, 32.29-31, 36.18-38.5, 39.4-43.9.

sciences that made possible the work in this style of harmonic theory which was later perceived as paradigmatic and authoritative, that is, the work of Aristoxenus.[7]

For Aristoxenus, melody, musical attunement and so on exist only in the audible domain. The principles governing their organisation are peculiar to them, and we have access to them only through a process of abstraction from what we actually hear. They are neither identical with nor derivable from principles applicable across a wider range, allegedly general principles of good order as such, like those conjured up by the devotees of Pythagorean mathematics. Aristoxenian principles are reached through reflection on the results of meticulous empirical observation, and what they specify is a φύσις in the Aristotelian sense, the φύσις of something that exists, as I have said, only in the realm of sound. Its essence and attributes can be discovered only through the study of its instances in that realm, since there are none anywhere else. They can no more be inferred from supposedly higher and wider principles than can those constituting the φύσις of a kiwi or a kookaburra.

All this has consequences for the ways in which musical items are conceptualised and the language in which they are described. It is nonsense, from an Aristoxenian point of view, to represent relations between pitches as ratios of numbers, since that is not how they manifest themselves in the auditory domain; and since they exist only as items in that domain, Pythagorean representations simply fail to describe them as they are. Aristoxenus argues that pitches are located in a dimension analogous to linear distance in ordinary space. Different pitches are different points in the 'space' inhabited by the objects of hearing, and the singing voice moves across this space from one point of pitch to another, just as we walk from point to point in the space occupied by tangible and visible objects.[8] The relations between pitches, then, are not ratios, but are the linear 'distances' between these points; it is in that sense that they are correctly described as διαστήματα, 'gaps', or 'intervals' as we now call them. More broadly, the language of his science is a fusion of regular musicians' terminology with that of Aristotelian physics, conceptually modified to fit the quasi-spatial model which he thinks best captures the manner in which the phenomena actually present themselves to perception, the manner, that is, in which they actually exist as music.

Remarks in the *Posterior Analytics* suggest that Aristotle regarded the two forms of harmonic science as complementary rather than as rivals;[9] but he is in this respect exceptional. Most participants and commentators treat them as flatly incompatible with one another, and not just on a broad conceptual or methodological level; they generate contradictory conclusions on matters of musical detail too. Thus, for example, various intervals that are important in any musical construction, such as the octave, the fifth, the fourth and the tone, can readily be divided, within the linear Aristoxenian model, into sub-intervals of equal sizes. The Pythagoreans, by contrast, had a mathematical argument to show that such equal divisions are impossible. For Aristoxenus, the octave is equal to exactly

[7] On the Aristotelianism of Aristoxenus' concepts and methods see especially A. Bélis, *Aristoxène de Tarente et Aristote: le Traité d'harmonique* (Paris 1986), and more briefly my comments in *Greek musical writings* vol. 2 (Cambridge 1989) 66-68, 123-24.

[8] See especially *El. Harm.* 8.13-12.34.

[9] *An. Post.* 75a38-b17, 78b34-79a6, cf. 90a14-23.

six tones; in Pythagorean mathematics it comes out as slightly but significantly less.[10] On the basis of the evidence of the ear, Aristoxenus recognises the interval of an octave plus a fourth as a genuine concord; Pythagoreans argue that it cannot be, since the ratio corresponding to it is not of the appropriate mathematical form.[11]

As I remarked at the beginning, no treatises in either tradition survive from the period which I label for convenience as 'Hellenistic', the period, that is, between the date of the Euclidean *Sectio Canonis* – which I place at about 300 BC[12] – and the early Roman empire. When we look at writings from the first couple of centuries of the imperial era and ask what picture they present of Hellenistic harmonic theorizing, the answer is to say the least confusing. Some sources convey the impression that during that period nothing more exciting had happened than some copying and summarising, along with a few minor adjustments around the edges. Thus in the methodological discussions of Ptolemaïs of Cyrene, Didymus ὁ μουσικός (probably in the time of Nero), Ptolemy in the early second century AD and a number of others, the major schools of thought identified are still 'Aristoxenian' and 'Pythagorean'.[13] Though the differences between them have been recast in the language and through the conceptual apparatus of post-Aristotelian philosophy, they are essentially much the same as before, and they are still sniping at one another across the barricades over much the same issues. The conception of 'Pythagoreanism' retailed in these surveys reflects very closely the contents and procedures of the *Sectio Canonis*, together with traces of some of its fourth-century predecessors; and the 'Aristoxenian' approach is simply that of Aristoxenus himself. Similarly, the scraps of Aristoxenian theory passed on by Vitruvius are a little garbled,[14] but they depend on no source but Aristoxenus or a summary of his writings; and the Aristoxenian compendium usually attributed to an otherwise unknown Cleonides, and dating perhaps from about 100 AD, is merely Aristoxenus in easy sound-bites, with only a very few trivial embroideries added by later followers.

Other sources, however, suggest quite another story, one in which the period gave birth to substantial, if rather eccentric work in harmonic theory. It produced, so far as we know, no very novel doctrines; but the perspective it adopts is significantly different from those of its fourth-century precursors. The linking thread is that its proponents seem implicitly or (less often) explicitly to reject the view that there is an unbridgeable gulf between mathematical and empirical harmonics, and their work combines concepts and procedures from both sides

[10] On the impossibility of dividing these intervals equally, see the theorem of [Eucl.] *Sect. Can.* proposition 3, applied to specific cases in propositions 16 and 18. The theorem is probably derived from Archytas, to whom Boethius (*Inst. Mus.* 3.11) attributes a very similar proof. On the size of the octave see *Sect. Can.* propositions 9 and 14.

[11] Aristoxenus *El. Harm.* 45.7-33. For the Pythagorean view see Ptolemy's critique at *Harmonics* 13.1-23. The *Sect. Can.* identifies the ratios of all the other concords within the regular two-octave span, but on the subject of the octave plus a fourth it is significantly silent.

[12] Its authorship and date are matters of continuing discussion. For a very thorough review of the evidence and an impressive attempt to resolve the problems, see A. Barbera, *The Euclidean division of the canon: Greek and Latin sources* (Lincoln 1991). I am grateful to Professor Barbera for his (largely favourable) comments on a paper, at present unpublished, in which I try to establish the fourth-century origins of all the material in the treatise, and the probability that the whole work was written at a date close to the beginning of the third century BC.

[13] For the relevant passages of Ptolemaïs and Didymus see n.3 above, with the addition of Porphyry *In Ptol. Harm.* 27.17-28.26. In his critiques of earlier theorists, the Pythagorean and Aristoxenian schools of thought are the only ones that Ptolemy mentions: see for instance *Harmonics* Book 1 ch. 2.

[14] Compare, for example, *De architectura* 5.4.2 with its Aristoxenian original, *El. Harm.* 8.13-10.10.

of the divide. In this respect their position has something in common with the one sketched by Aristotle in the *Posterior Analytics*, though they nowhere come close to a programme as tidy and systematic as his. I shall split them into two groups. One of them, broadly Peripatetic in flavour and most clearly exemplified in the Aristotelian *Problems* (Books 11 and 19), blends mathematical and empirical approaches, but – rather mysteriously – shows no awareness whatever of the work of Aristoxenus. In so far as the language and concepts of its empirical strand are of a technical sort at all, they go back to Aristotle himself and beyond him to Plato. These writings are not altogether isolated curiosities, but for present purposes I shall leave them on one side. In the other and much more extensive group of sources the predominant angle of attack is mathematical or 'Pythagorean'; but it has appropriated, usually without acknowledgement, a considerable repertoire of concepts, definitions and modes of analysis which have unquestionably been imported from Aristoxenian territory. As a tool of musical analysis, Aristoxenus' approach was conceptually much richer than that of the Pythagoreans, and far more sensitive to the variations and interrelations of structure that could be manifested in real music-making. The authors I have in mind seem to have thought that if they eliminated a handful of flat contradictions between Aristoxenian findings and the conclusions of the mathematicians, a great deal of the musical largesse which his work provided could be absorbed and reinterpreted within the framework of Pythagorean mathematics.

All the texts on which I base these remarks were written later than the Hellenistic period, and they are by no means meticulous in naming their sources. When they do, the sources they name are rarely earlier than the first century AD, though there are a few helpful exceptions. If what they found in their immediate authorities really does reflect the work of still earlier, Hellenistic writers, then, we seem to be left largely in the dark about who those writers were and in what contexts they were pursuing their activities. One group of candidates can be ruled out from the start. There is no evidence and no probability that they were among the producers of Pythagorean pseudepigrapha of the sort collected by Thesleff,[15] since these seem wholly innocent, as a genre, of the scientific and mathematical technicality tied up with harmonic theory. Some of the scattered remarks they make on musical topics have a weird sort of charm, but they have nothing to do with the subject we are addressing. Again, we might suppose that the material got into our surviving sources from writings by some eclectic group of Hellenistic scientists based, perhaps, in Alexandria, and that these writings amounted to free-standing treatises in harmonics, comparable to those known from the fourth century and from the imperial period. But that would be the merest speculation; there is not a scrap of evidence to confirm that such treatises or such scientists existed. The hypothesis for which I shall argue in the rest of this paper is that the vehicles in whose capacious luggage-compartments this material was carried consisted, either wholly or in very large part, of studies expounding and discussing the dialogues of Plato, especially the *Timaeus*, and most specifically, of course, the passage on the structure of the World Soul. Whether or not these studies can properly be described as 'commentaries' is debatable, but it is an issue I shall simply ignore.

The first point I want to make is a general one. If we temporarily exclude Porphyry (in his commentary on Ptolemy's *Harmonics*), who is deliberately collecting illustrative citations

[15] H. Thesleff, *The Pythagorean texts of the Hellenistic period* (Abo 1961).

from wherever he can find them, the texts which rehearse most extensively these confusing combinations of Aristoxenian and Pythagorean materials are all ones in which discussions of the *Timaeus* play a central role. They include writings by Thrasyllus, Adrastus, Theon of Smyrna, Nicomachus and Plutarch. Adrastus' contributions, according to Porphyry, come from his Εἰς τὸν Τίμαιον;[16] Theon was offering a compendium of 'mathematics useful for reading Plato'; Nicomachus' most relevant work is his Ἁρμονικὸν ἐγχειρίδιον, an essay that pivots around the examination of the *Timaeus* that occupies its eighth chapter; and the crucial piece by Plutarch is his *De procreatione animae in Timaeo*. The only doubtful case is the earliest, Thrasyllus. The work from which the excerpts that concern us are taken was probably called Περὶ τοῦ ἑπταχόρδου, though there is some doubt about that,[17] and none of the excerpts touches on the *Timaeus* directly. But Thrasyllus' interest in Plato is well known, and statements made by Theon show that he put his harmonic analyses to use in an examination of the 'music of the κόσμος'.[18] This part of Thrasyllus' work has not survived, but it seems overwhelmingly probable that the music in question is either that of the *Timaeus* or that of the Myth of Er in the *Republic*, perhaps both.

There are enough similarities between the procedures and agendas of these works to make it clear that they have a common background of some sort. On the other hand, they neither depend, as a group, on some single, unmediated source from an earlier period, nor are they merely echoing one another. Despite the shared features, when it comes to the details of their selections of materials, choices of definitions, modes of presentation, patterns of argumentation and so on, there are far too many disparities between related portions of these texts for a hypothesis of that sort to hold water. Thrasyllus, in particular, is too eccentric a member of the group to be the original of which the others are reflections, as he would have to be if the 'echo' hypothesis were to stand, since he is the earliest by at least half a century.

The common ground between them lies in the series of topics which they seem to treat as necessary preliminaries to their central investigation, that is, so I believe, to an examination of the constructions of the *Timaeus*. It may seem odd to characterize Nicomachus' contributions as 'preliminaries', since though one regular element of the programme appears early in his treatise, most of it is bundled together right at the end, in Chapter 12. But his essay advertises itself, both at the start and at the finish, precisely as a propaedeutic to a larger-scale work on the same subject;[19] and statements in Chapter 11 indicate that the promised work will itself culminate in a 'division of the κανών ... in the manner of Timaeus of Locri, whom Plato also followed'.[20]

One major part of the shared programme consists in a series of definitions of basic musicological concepts such as 'note' (φθόγγος), 'interval' (διάστημα), 'scale' or 'system' (σύστημα) and so on, together with summary accounts of their differentiae and expositions of the structures of the three principal 'genera' of harmonic organisation. This repertoire of topics is very plainly borrowed from Aristoxenus. To it is added, however, another group of

[16] Porphyry, *In Ptol. Harm.* 96.1-2.

[17] See Porphyry, *In Ptol. Harm.* 91.13-14, 96.16, with Düring's notes and apparatus criticus.

[18] See Theon Smyrn. 204.23-205.6.

[19] Nicomachus, *Enchiridion* 238.6-12, 265.5-8 Jan. It is probably this larger work that forms the basis of the first four books of Boethius, *Inst. Mus.*

[20] *Enchiridion* 260.16-17.

topics which are equally obviously not Aristoxenian at all; these typically include a review of Pythagorean theories about the physics of sound and especially of pitch, a survey of the means by which Pythagoras, or Pythagoreans, are alleged to have discovered and confirmed the ratios of the musical concords, certain rather special conceptual distinctions focussing – without saying so directly – on points at which Pythagorean and Aristoxenian approaches diverge, and a discussion – usually, this time, explicitly naming the two traditions – of another special question at issue between them, that is, whether it is or is not possible to divide the interval of a tone into equal parts.

Traces of all or most of these topics can be found in each of the sources I have mentioned, generally packed closely together in a more or less continuous sequence.[21] The authors do not always present them in the same order; not all of them define exactly the same set of basic items; their definitions do not always agree; and some of them deal only perfunctorily with matters on which others go into elaborate detail. But the general pattern is consistent enough to be recognised. Another feature they have in common is an alarming propensity to certain idiosyncratic brands of confusion or ineptitude. Thus the accounts that they retail of the reasons why a tone cannot be equally divided repeatedly advance considerations of breathtaking mathematical naïvety, and make it clear also that the authorities on which they depend had no access to or no understanding of the sophisticated proofs presented by Archytas and again in the *Sectio Canonis*.[22] Their 'dumbed-down' versions must have crept into the tradition by a different route. The 'special conceptual distinctions' which I mentioned incorporate muddles to which I shall return in a moment. The majority of the Pythagorean 'experiments' with instruments which they report do not work, and these sources rarely give special prominence to the one or two that do.[23] And to return to my original theme, the definitions and classifications from which they set out regularly include ones drawn from Aristoxenus, which cannot coherently be incorporated, just as they stand, into the Pythagorean framework of their overall positions.[24] Similarly, most of them rehearse an Aristoxenian account of the genera, enharmonic, chromatic and diatonic, without even trying to recast them in terms of Pythagorean ratios;[25] Thrasyllus, who does make the attempt, gets

[21] See for instance Thrasyllus ap. Theon Smyrn. 47.18-49.5; Adrastus ap. Theon 49.6-62.4, 63.25-72.20 (some of the material in these passages appears to have been inserted by Theon himself); Nicomachus, *Enchiridion*, especially chs. 6 and 12; Plutarch, *De procr. an.* 1020e-1021d.

[22] Eg. Theon 53.8-16, 69.12-70.3, 70.14-19 (probably from Adrastus), Plutarch, *De procr. an.* 1021c-e. Nicomachus (*Ench.* 263.24-264.5) merely asserts that the 'semitones' involved in the construction of the octave are not exact halves of a tone, and reserves his 'demonstration' of the point for the larger work which he promises one day to produce.

[23] See especially Theon 56.9-61.11, Nicomachus, *Ench.* ch. 6. Of the devices which were used, according to these passages, to 'demonstrate' the ratios of the concords, the only ones that are tolerably reliable are those in which the terms of the ratios correspond to lengths of a stretched string, as on the monochord. Nicomachus mentions monochords merely as one item in a list whose other members inspire no confidence, and Theon (or Adrastus) devotes a few lines to a similar device. But the bulk of their discussions describe procedures which either fail in principle (as most notoriously Nicomachus' story of Pythagoras and the 'harmonious blacksmith'), or else are in principle acceptable, so far as the laws of physics are concerned, but in practice are hopelessly inaccurate (see Ptolemy's comments on many of these devices at *Harm.* 16.32-17.20).

[24] See for instance, the definitions of διάστημα (interval) given by Nicomachus (*Ench.* 261.8) and Plutarch (*De procr. an.* 1020e), which are wholly Aristoxenian in inspiration, and cannot be accommodated to a representation of intervals as ratios.

[25] Eg. Adrastus ap. Theon 53.17-56.5, Nicomachus *Ench.* 262.7-263.17.

into a terrible tangle with the chromatic and doesn't even pretend to be able to map the Aristoxenian enharmonic's quarter-tones onto a Pythagorean 'division of the κανών'.[26]

Quite generally speaking, in fact, the Aristoxenian elements in these writers' work have not been convincingly remodelled to fit them into their new conceptual context. There are a few honourable exceptions, of which the second chapter of Nicomachus is a notable example. It takes Aristoxenus' picture of the two ways in which the voice 'moves' in the dimension of pitch – a picture which is wholly incompatible, in its original form, with the Pythagorean model – and by substituting a different terminology and replacing Aristoxenus' nexus of metaphors with others, it succeeds simultaneously in capturing the substance of the original distinctions and in tying them in consistently with the apparatus of Pythagorean physics. It contains, I think, one momentary lapse, when the implications of the phrase ἐπὶ παντὶ φθόγγῳ ἱστάμενον (238.23), if taken seriously, would push us back for an instant into an Aristoxenian mode of thought. Otherwise the transformation is complete, and its author – whether it is Nicomachus himself or one of the 'Pythagoreans' to whom he attributes it (238.18-19) – has evidently understood the conceptual problems rather well and has tackled them cleverly. But this level of understanding and sophisticated linguistic and conceptual manipulation is, as I have said, very much the exception.

None of these authors tells us directly where the programme of topics they address or its components come from. The nearest we find to an explicit attribution is in Plutarch. 'What the λεῖμμα is and what is Plato's meaning you will perceive more clearly, however, after having first been reminded briefly of things that are regularly said in the Pythagorean σχολαί' (1020e). Here the λεῖμμα is the small interval 'left' at the bottom of a diatonic tetrachord, and it is of course an element in the structure of the World Soul;[27] and Plato's 'meaning' (διάνοια) is the intended significance of his account of that musical-cum-mathematical structure. But what are the 'Pythagorean σχολαί' which contain the preliminary material that Plutarch now rehearses, material corresponding rather closely, though there are omissions and divergences, to the expositions in Thrasyllus, Adrastus, Theon and Nicomachus? Cherniss, in his Loeb translation, renders σχολαί simply as 'treatises'. But they are certainly not what this rendering seems to suggest – that is, independent compendia of authentically Pythagorean harmonic theory – if only because the very first proposition that Plutarch offers under this heading, a definition of 'interval', διάστημα, is unambiguously Aristoxenian and in mathematical harmonics would be unintelligible. Further, the sequence of propositions and arguments that follows makes no sense as a general introduction to harmonics; it seems to be designed precisely as a route towards an analysis of the *Timaeus* passage. The selection and arrangement of materials can

[26] Thrasyllus ap. Theon 90.22-93.9. His one statement about the enharmonic is at 92.27-93.2; at best it fixes only the upper boundary of the interval into which the two quarter-tones must be fitted. The problems introduced by his treatment of the chromatic are too complex to be explored here; they arise from the fact, not mentioned by Thrasyllus, that the small interval called the λεῖμμα (whose ratio is 256:243) is less than half a tone.

[27] *Timaeus* 36b2-5, where the noun λεῖμμα does not itself occur, but the sense of a 'remainder' is conveyed by two instances of the related verb λείπειν. The first surviving author to use the noun in this musicological sense seems to be Adrastus (ap. Theon 68.11-12), but his reference to this interval as 'the so-called λεῖμμα' indicates that the usage was current before his time. It may well have been coined by a writer seeking to encapsulate the idea introduced by Plato's verbs in the *Timaeus*. From the time of Adrastus onwards it becomes part of the standard vocabulary of mathematical harmonics.

hardly be due entirely to Plutarch himself, in view of the parallel sequences in our other sources. It seems more likely, I suggest, that the Pythagorean σχολαί to which Plutarch refers are direct precursors of the passages we find in Thrasyllus, Adrastus and Theon, in the sense that they too were written as introductions to those aspects of harmonic theory which are 'useful for reading Plato'. Plutarch does not call these introductory passages 'commentaries on the *Timaeus*' because they do not deal directly with that text, but serve only as preliminaries to its exegesis. There are at least three possible reasons why they are called 'Pythagorean', despite their loose assimilation of Aristoxenian ingredients. One is that the *Timaeus* itself was associated very closely with Pythagorean thought, and its analysis of the World Soul is wholly in the style of that harmonic tradition. Secondly, the principal allegiance of the commentators who assembled these introductions is also to harmonics in its 'Pythagorean' guise, and could hardly have been otherwise if it was to unravel Plato's text. It is the Aristoxenian ingredients that are intrusive. Thirdly, while their borrowings from Aristoxenus are generally tacit, they advertise their Pythagorean credentials enthusiastically. On the occasions when they register disagreements between the two brands of harmonics, notably over the issue of the equal division of the tone, they always come down explicitly on the Pythagorean side; and in several cases they follow the convention of attributing views or procedures of which they approve to Pythagoras himself or to one of his early and eminent disciples.

It is tempting to suppose that Plutarch found these 'Pythagorean discussions' among the preliminaries set out in the studies of the Timaean World Soul by Hellenistic authors whom he names directly. They would fit well enough with what he tells us, in particular, about Crantor, Clearchus and Theodorus, especially at 1022c ff. But that is a guess we are not in a position to confirm. In order to trace aspects, at least, of this material back to an identifiable source, we shall have to take a different route, and we shall need to follow up one or two issues in a little detail. I shall focus initially on our authors' definitions of the term διάστημα, 'interval', and on some patterns of discussion that are repeatedly linked to them.

The definition given by Plutarch (1020e), πᾶν τὸ περιεχόμενον ὑπὸ δυοῖν φθόγγων ἀνομοίων τῇ τάσει, 'all that which is bounded by two notes differing in pitch', is unmistakeably Aristoxenian, as I have said. In Aristoxenus' own formal definition at *El. Harm.* 15.24 ff, the operative word is ὡρισμένον rather than περιεχόμενον, but the latter usage is also common in his writings, as for instance in his definition of ἀσύνθετον διάστημα, 'incomposite interval', at 60.10-11. It reappears in the definition of διάστημα provided at the beginning of the thoroughly Aristoxenian handbook of Cleonides[28] and in a number of other similar sources. An interval, under this conception, is a space bounded or 'contained' by two different points of pitch. With this we may contrast the definition attributed to Thrasyllus by Theon at 48.8-10, διάστημα δέ φησιν εἶναι φθόγγων τὴν πρὸς ἀλλήλους ποιὰν σχέσιν, 'he says that a διάστημα is a specific relation (σχέσις) in which notes stand to one another'. At a casual glance that may sound vague, but in its intellectual context it is not; it is a direct reflection of Euclid's definition of λόγος, ratio, at *Elements* Book 5 definition 3, as δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικότητά ποια σχέσις, abbreviated by Theon himself (73.16-17) as δυοῖν ὁρῶν ὁμογενῶν ἡ πρὸς

[28] Cleonides, *Eisagoge* 179.11-12.

ἀλλήλους ποιὰ σχέσις. Thrasyllus, then, in sharp contrast to Plutarch's source, is representing 'interval' in the guise of 'ratio', in an appropriately Pythagorean manner.

Consider now the account in the twelfth chapter of Nicomachus. διάστημα δ' ἔστι δυοῖν φθόγγων μεταξύτης, it begins (261.8). That sounds Aristoxenian; an interval is not a ratio but what 'lies between' two notes. But now Nicomachus, like Thrasyllus, introduces the notion of σχέσις. σχέσις δὲ λόγος ἐν ἑκάστῳ διαστήματι μετρητικὸς τῆς ἀποστάσεως, 'a relation is the ratio which measures the distance, ἀπόστασις, within each interval' (261.8-10). It looks as if Nicomachus is trying to have his cake and eat it - to retain the concept of a διάστημα as a distance between two points, and at the same time to insist that this distance is to be 'measured' as a ratio, that is, not as a distance at all. Something rather similar happens at Theon 81.6 ff, where once again διάστημα is defined in terms of τὸ μεταξύ, 'what lies between'; but here what we find next is not the statement that a διάστημα is measured by its ratio. It is a discussion of the difference between a διάστημα and a λόγος.

With this we are fairly launched upon a topic which resurfaces repeatedly in authors relevant to our investigations. Attempts to draw such a distinction appear in Theon, Nicomachus and Thrasyllus, and in a string of authors cited by Porphyry in his commentary on Ptolemy's *Harmonics*[29], of whom at least three, Demetrius, Panaetius and Aelianus, enunciated it in the context of studies of the *Timaeus*. Porphyry notes that point explicitly for Demetrius and Panaetius in the passage I have just mentioned, and he quotes the relevant statements from Aelianus earlier in his commentary, at 35.13 ff, as coming from the second book of his Εἰς τὸν Τίμαιον ἐξηγητικά.

Confusion is endemic in these discussions. To begin with, there are unhelpful shifts of terminology. Sometimes the distinction is between λόγος and διάστημα, 'ratio' and 'interval', as in Theon. Sometimes the term contrasting with λόγος is not διάστημα but διαφορά, 'difference'; and sometimes it is ὑπερβολή or ὑπεροχή, 'excess', or ὑπερβολή ἢ ἔλλειψις, 'excess or deficiency'. Several authors use a mixture of these terms, and it is not always clear whether they mean them to be interchangeable. Thus in Aelianus[30] διάστημα and διαφορά are equivalents; in Nicomachus a διαφορά is the ὑπερβολή ἢ ἔλλειψις φθόγγων πρὸς ἀλλήλους,[31] but he does not say that this 'excess or deficiency' is the same as 'that which lies between two notes', the διάστημα, and we cannot confidently decide whether he means that or not. Thrasyllus, at the relevant point in his discussion, defines διάστημα in terms of διαφορά, but when he goes on to draw a contrast with λόγος it is between λόγος and ὑπεροχή;[32] and nothing in the sequel, as we have it, confirms that this ὑπεροχή is, or that it is not, identical with the διάστημα or διαφορά, since neither of these terms reappears.

If these obscurities were confined to a terminological level they might not be very important. But in fact the confusion runs deeper. A musical διάστημα, in Aristoxenian terms, is a distance between two notes. In Pythagorean theory such an interval is represented

[29] Theon 81.6-82.5, Nicomachus, *Ench.* 261.8-18, Thrasyllus ap. Porphyry, *In Ptol. Harm.* 91.13-92.8, Porphyry *In Ptol. Harm.* 91.4-93.4.

[30] Ap. Porphyry, *In Ptol. Harm.* 35.15-17.

[31] *Ench.* 261.10-11.

[32] Porphyry, *In Ptol. Harm.* 91.16-92.8.

as a ratio, λόγος; the ratio of the octave is 2:1, that of the λεῖμμα is 256:243, and so on. Very commonly, as for instance in the *Timaeus* and the *Sectio Canonis*, the word διάστημα itself appears where we might expect λόγος; it is διαστήματα that are described as duple (ie. 2:1), hemiolic (3:2), epitritic (4:3) and the rest. Now between the terms of each of these ratios there is a 'difference', διαφορά, or to put it differently, they stand to one another in a relation of 'excess' and 'deficiency', ὑπεροχή and ἔλλειψις. But of course the 'difference', in this sense, between the terms of the ratio of the λεῖμμα, for example, 256 and 243, is not at all the same as the interval, διάστημα, between the notes these terms represent. There is no sense in which the interval is the number 13, even though unspecified writers mentioned by Theon and Plutarch seem to suppose that there is.[33] To compound the problem, the difference between the numbers can itself be described as a διάστημα, a 'distance', especially when the numbers are conceived as identifying different points on the string of an experimental instrument such as the monochord. If one length of string is 9 units long and another is 6 units long, the ratio between the lengths is 3:2. The 'interval', διάστημα, between the notes they sound is a perfect fifth; but the 'distance', also διάστημα, between their end-points on the string is 3 units. Hence when Theon, for instance, argues that λόγος differs from διάστημα, since the ratio 2:1 is distinct from the ratio 1:2, whereas the διάστημα is in each case the same,[34] we cannot tell whether he means that the musical interval is in each case the same, an octave, or that the difference between the two terms is the same, 1, or that the distance between the end-points of relevant lengths of string is the same, or indeed whether he imagined that all these propositions were equivalent. Similar ambiguities infect the treatments of Nicomachus and Aelianus;[35] and though Thrasyllus in his discussion of these distinctions uses the word διάστημα to refer unambiguously to a musical interval, in his division of the κανών it has to be read, equally unambiguously, as indicating a distance along the instrument's string.[36]

All this is very vexing; but it offers a helpful clue to someone engaged in historical detective-work, since one can trace the footsteps of these confusions back into the misty past beyond Thrasyllus. They take us, in fact, right back to the third century; and the miscreant to whose door they lead is Eratosthenes. Relevant citations from Eratosthenes appear in Theon and Porphyry,[37] as well as in various later writers. They incorporate, inter alia, exactly the same argument and the same ambiguity in the use of διάστημα as is found in Theon's own presentation; and Porphyry in fact complains that Eratosthenes fails to make it clear what he means by the term – 'he does not establish either what he means by 'διάστημα', or in what respect it differs from λόγος' (91.9-10).

[33] Theon 69.3-6, Plutarch, *De procr. an.* 1022a; cf. the views attributed (probably wrongly) to Philolaus at Boethius, *Inst. Mus.* III.5 (DK44 A26).

[34] Theon 81.6-16.

[35] Nicomachus, *Ench.* 261.8-18, Aelianus ap. Porphyry, *In Ptol. Harm.* 35.13-22.

[36] For his discussion of the distinctions see Porphyry, *In Ptol. Harm.* 91.13-92.8; for the relevant part of his division of the κανών see Theon 88.1-10.

[37] The most important, from the present perspective, are at Theon 81.17-82.5, Porphyry, *In Ptol. Harm.* 91.4-10. Compare also Porphyry 92.24-5; other citations from Eratosthenes which, though less immediately relevant, nevertheless have a bearing on these issues are at Theon 82.22-84.6, 107.15-22.

Ambiguity of expression is of course not the same as actual confusion. But the hypothesis that Eratosthenes really was confused about this issue is spectacularly confirmed in another source; and the evidence of that source casts further light on the agenda of Hellenistic harmonics. Before we review it I should point out that the relevant Eratosthenian material seems, like those of the later treatises, to have been embedded in a discussion of issues in Plato. Theon identifies one of his quotations as coming from Eratosthenes' *Platonikos*,[38] and it has been persuasively argued by Klaus Geus that all the surviving testimonia on Eratosthenes' musicology derive from that source.[39] Their context can hardly have been anything but a discussion of the *Timaeus*, and particularly of its account of the World Soul. If that is correct, as I think it is, at least some of the *Timaeus*-related material we have been studying will have been run to earth in a similar environment in the Hellenistic period.

Now to the other source I mentioned, which is Ptolemy in *Harmonics* II.14. Ptolemy, whether deliberately or not, adopts a linguistic strategy which enables him to keep clear of the conceptual bramble-patch in which others had become entangled; quite extraordinarily, given his topic, he avoids the word διάστημα altogether, except in non-musical contexts and in discussions of Aristoxenus himself. (I have located only two exceptions to this rule, in a treatise running to some 120 pages.) What Ptolemy attributes to Eratosthenes is a group of 'divisions of the κανών', that is, sets of ratios specifying the relative string-lengths, on an instrument akin to the monochord, that will yield the intervals of systems in each of the three genera, diatonic, chromatic and enharmonic.[40] Eratosthenes' diatonic division is unremarkable; it corresponds directly to that involved in the *Timaeus* and in many later sources. Given its presumably Platonic setting, it would have been surprising if he had put anything different in its place. The other two divisions have several unusual features; but the most striking thing about them is that when translated into lengths of string measured on the 120-unit scale that Ptolemy uses, the lengths they specify are identical with those that Ptolemy presents, in the same passage, as producing the enharmonic and tonic chromatic systems of Aristoxenus.[41]

The correspondence between Eratosthenes' scheme and that attributed to Aristoxenus cannot be coincidental. The sets of ratios involved are unique to Eratosthenes, and none of the many other divisions suggested in antiquity will give the same result. The figures can therefore be construed as evidence of a deliberate attempt to translate Aristoxenus' analyses out of the language of linear intervallic distances into that of Pythagorean ratios. Here, then,

[38] At 2.3 and 81.17.

[39] K. Geus, 'Anmerkungen zur "Musiktheorie" des Eratosthenes', in K. Döring, B. Herzhoff, G. Wörle, eds, *Antike Naturwissenschaft und ihre Rezeption* (Trier 1995). Eratosthenes' contributions to musical theory are the subject of a chapter in a forthcoming thesis by David Creese (Birmingham); see also a short article, A. Barker and D. Creese, 'Eratosthenes', forthcoming in *Die Musik in Geschichte und Gegenwart* (Kassel 1999-), Personenteil vol. 5.

[40] Most of the relevant text of II.14 was lost before the Byzantine era; it was reconstructed by Isaak Argyros in the 14th century and revised by John Wallis in the 17th. For details see Düring's introduction to his edition of Ptolemy, and notes ad loc. in J. Solomon, *Ptolemy, Harmonics: translation and commentary* (Leiden 2000). This may be thought to raise problems about the authenticity of the chromatic and diatonic divisions attributed to Eratosthenes in the MSS dependent on Argyrus (but not about his enharmonic, which is described in a surviving passage of the original, at 71.3-4); but since the tables of ratios and string-lengths survived in the pre-Byzantine MSS (though in variously confused forms), these problems are not acute.

[41] This will become clear from an inspection of the tables in *Harm.* II.14.

probably in the context of a *Timaeus*-discussion, we find Eratosthenes engaged in a project of just the sort we were looking for, that of absorbing into the framework of mathematical harmonics at least some of the riches of Aristoxenian musicology. But the task of translation is not straightforward. There is no direct way of deciding, for example, what ratio corresponds to one of the quarter-tones of Aristoxenus' enharmonic division, since no ratio of integers, compounded with itself four times, will yield the ratio of the tone, 9:8.

In the previous chapter, II.13, Ptolemy explains the basis on which he has assigned determinate string-lengths to the pitches of Aristoxenian systems. His purpose is to make it possible to compare them directly with the divisions proposed by other authorities, including Eratosthenes, all of whose divisions are presented in terms of ratios and can unproblematically be mapped onto relative lengths of string. Now if the procedure by which he converted Aristoxenus' systems into comparable terms were one that Ptolemy had devised himself, my reconstruction of Eratosthenes' intentions would be seriously undermined, since there would be no adequate reason to assume that he would have assigned string-lengths to Aristoxenus' constructions in precisely the same way. But I find it unbelievable that the procedure is Ptolemy's own. Whatever else he was, he was a highly skilled mathematician; and the procedure in question is mathematically absurd. I shall not analyse it in detail. The essential point is simple; it involves just the same confusion between musical intervals and distances along a string as we found in Thrasyllus and Aelianus, and which were generated out of ambiguities lurking also in the discussions of Theon, Nicomachus and, most significantly, Eratosthenes himself. It in fact aligns equal intervals in Aristoxenus' divisions with equal 'distances' or 'differences' between the lengths of string sounding the notes that bound them. The procedure is hopelessly inept, as a simple example will show. In Aristoxenus' enharmonic, there are two equal quarter-tones at the bottom of each tetrachord. In Ptolemy's reconstruction, the lowest note of one tetrachord is sounded by a string 80 units long. The next note up is assigned a string two units shorter, 78, and by the principle that equal distances go with equal musical intervals, the length of the next differs again by two units, giving 76. The three strings bounding the two quarter-tones are thus 80, 78 and 76 units long. But the intervals between the notes they sound will obviously not be equal, as Aristoxenus intended. In Pythagorean terms, the first has the ratio 40:39, while that of the second is 39:38; and the interval represented by this second ratio is larger.[42] As I said before, the procedure is ridiculous, and Ptolemy knew that it was. We do not have to rely on his general credentials as a mathematician to be sure of that fact. In a bad-tempered passage of Book I he identifies and criticises precisely the same absurdity, and accuses Aristoxenus – quite falsely – of committing it, and so falling into egregious error.[43] Why then does he rely on this indefensible procedure in Book II?

I can find only one hypothesis that will explain the facts. Ptolemy had access to some source in which Aristoxenian systems were translated, by this procedure, into relative lengths of string. He noted the ineptness underlying it, in Book I, but both there and in Book II he made the mistake of fathering on Aristoxenus himself the supposition that underlies it, that is, that equal musical distances, διαστήματα, correspond to equal geometrical διαστήματα

[42] The difference may seem negligible, and no doubt from an aesthetic perspective it is. But these are precisely the sorts of mathematical minutiae to which Ptolemy attaches importance.

[43] *Harm.* 20.23-21.8.

between the end-points of lengths along a string. In that case he could have thought himself justified both in criticising the supposition in one passage and nevertheless also in using it for his own purposes in another, since he would on this interpretation be setting out Aristoxenus' divisions in the manner in which Aristoxenus himself conceived. This hypothesis accounts for what we find in the text; and it has the advantage that it does not attribute to Ptolemy a mathematical blunder or a dishonest exploitation of reasoning that he knew to be faulty. It merely convicts him of a historical error.

If my interpretation is anywhere near the mark, it is overwhelmingly probable that the source for all this is Eratosthenes. It is in Eratosthenes that we first find the ambiguities that can prompt confusion between the various senses of διάστημα, διαφορά and the rest; and it is precisely on the basis of such confusions that Eratosthenes' divisions can be read as translations of those of Aristoxenus. We may note here that Ptolemy probably did not have direct access to Eratosthenes' treatise, since he mentions him nowhere else. The information he found is likely to have been transmitted by a certain Didymus, probably to be identified as a musical theorist at work in the time of Nero, whom Ptolemy discusses at length, and from whose work, so Porphyry alleges, a great deal of Ptolemy's material is derived.[44] If that guess is right, it helps to tie the route of transmission together in another way too, since Didymus' own divisions of the κανών, which Ptolemy records in the same passage, also show signs of having been designed – though in a different way – to reformulate the Aristoxenian constructions in terms of ratios. I have argued this thesis elsewhere[45] and will not pursue it now. But assuming that it is correct, my 'Aristoxenian' reading of Eratosthenes' divisions will evidently do something to explain why Didymus thought them worth recording. Perhaps he presented his own sets of ratios explicitly as providing a better translation of Aristoxenus than did those of his learned predecessor.

What we have found in Eratosthenes, then, is this. Somewhere in his reflections on Plato, probably on the *Timaeus*, was a discussion of the differences between λόγος and διάστημα, which also made use of the terms διαφορά, ὑπεροχή and ἔλλειψις. It involves serious ambiguities, and the opportunity for large-scale confusion. Just the same mode of discussion, the same ambiguities and sometimes clear instances of the relevant confusions reappear in a bundle of writers from early imperial times, all of whose work focusses in one way or another on Platonic exegesis in general and on the *Timaeus* in particular. The other feature that links these later writings is their idiosyncratic appropriation into mathematical harmonics of contributions from Aristoxenian theory. Eratosthenes, similarly, appears to be responsible for a striking, if mathematically flawed attempt to recast Aristoxenian analyses in Pythagorean terms, an attempt which is itself dependent on confusions of the sort which are rife in the later Timaean tradition. It is hard to imagine that it was a wholly isolated exercise. Eratosthenes presumably made other forays across the conceptual and methodological frontier, if only to provide his 'translated' *divisiones canonis* with a context and a rationale. It is worth noting that several of the later writers on our list also draw on Eratosthenes' work in passages we have not examined here. Adrastus, in his commentary on the *Timaeus*,

[44] Comments on Didymus occupy almost the whole of Ptolemy, *Harm.* II.13, and his divisions are recorded in the tables of II.14. For Porphyry's allegation see *In Ptol. Harm.* 5.11-14.

[45] 'Greek musicologists in the Roman Empire', in T. Barnes, ed., *The Sciences in Greco-Roman Antiquity*, *Apeiron* 27.4 (1994), 53-74.

exploits Eratosthenes' theory of 'elements' and elaborates his work on proportions, ἀναλογίαι; Nicomachus uses the same material and probably more in his *Introduction to Arithmetic*; and so does Theon, who is also among a squad of authors who discuss his curious poem *Hermes*, on the harmony of the spheres. There are other reports about his work in harmonic theory, unfortunately bereft of any context, in the so-called *Excerpta Neapolitana*, an anonymous compilation of uncertain date; and reports like those in the second-century texts resurface again, as is well known, in later writings on the *Timaeus*, notably in Proclus and Calcidius.

It appears, then, that Eratosthenes in his studies of Plato was at any rate one of the mould-breakers who sought to demolish the barricade erected in the fourth century between mathematical and empirical harmonics, and that in later times his influence was felt most strongly in writings that concerned themselves with the exegesis of musicological matters relevant to a study of the *Timaeus*. I cannot prove that his programme was picked up and developed by other Platonic commentators who were at work between his time and that of Thrasyllus, though I think it exceedingly likely. There is no evidence, at any rate, that strategies of this sort, which so heavily influenced the imperial-period *Timaeus* commentators, were pursued in this form anywhere, in Hellenistic times, except in the *Timaeus* tradition itself; and that would remain true even if Eratosthenes turned out to be its only relevant representative. I mentioned earlier that some curiously old-fashioned writings by post-Aristotelian Peripatetics also show signs of attempting a rapprochement between the two warring factions; but their project was quite differently conceived and made virtually no detectable impact on later harmonic theory. Similarly, once the frontiers had again been formally sealed off by methodologically sophisticated authors of the first and second centuries AD, it is only in writings linked to Platonism that we find significant traces of an ambition to bridge the divide; they include not only the ones we have looked at in this paper, but also the eccentric *De Musica* of the quasi-Neoplatonist Aristides Quintilianus, probably dating from the third century, whose first book is entirely Aristoxenian, but whose third adopts a conceptual apparatus derived equally unambiguously from the Pythagorean repertoire. Attempts at setting up a fruitful dialogue across the theoretical chasm constituted, so far as I can see, the most significant original contribution made by Hellenistic writers to the enterprise of harmonic science. Soberly considered they must be reckoned a failure, though perhaps a gallant one; but the bulk of the responsibility for setting them in motion and for carrying them forward into later centuries, whether the fact is to their credit or not, seems to fall squarely on the shoulders of early commentators on the *Timaeus*.[46]

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